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EFFECT OF RADIATION SCATTERING ON THE MELTING AND SOLIDIFICATION OF A FLAT LAYER OF A TRANSLUCENT MEDIUM

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The paper considers the effect of isotropic and anisotropic scattering of radiation on the melting (solidification) of a flat layer of a semitransparent medium between opaque surfaces. The mathematical model of the phase transition is the classical formulation of the Stefan problem. From results of numerical calculations it follows that radiation scattering has a significant effect on the rate of propagation of the phase transition front and formation of a temperature profile during melting (solidification) of a semitransparent medium.

Investigation of the complex radiative-conductive heat transfer (RCHT) during melting and solidification of semitransparent materials covers a wide range of temperatures — from low temperatures (for example, melting of ice by solar radiation) to high temperatures (for example, growth of crystals from melts or production of semitransparent materials). Because semitransparent media are highly transparent to thermal radiation in particular spectral regions, experimental studies of temperature fields in the bulk of a semitransparent material seem difficult. Therefore, investigation of the effect of thermal radiation on the formation of temperature fields and heat fluxes during melting and crystallization of a semitransparent material is of great practical significance. In this connection, numerical studies of the radiative-conductive heat transfer during phase transition of a semitransparent medium is an urgent problem. At present, have been few papers on this problem (see, e.g., [1]). Golova and Rubtsov [2] studied the effect of isotropic scattering of radiation on the melting of a semitransparent medium. Oruma et al. [3] investigated the effect of anisotropic scattering in a two-phase region on the rate of melting (solidification) of a semi-infinite semitransparent medium using a generalized model of phase transformation.

In the present paper, we study numerically the formation of a temperature field and radiant fluxes during melting and solidification of a flat layer of a semitransparent medium of thickness L which is between opaque, diffusely radiating and reflecting surfaces (Fig. 1). With satisfaction of the condition of local thermodynamic equilibrium and in the absence of convection, the Stefan problem in the classical formulation with constant thermal properties is written in dimensionless form

$$\begin{aligned} c_1\,\frac{\partial\theta}{\partial\eta} &= \Lambda_1 N\,\frac{\partial^2\theta}{\partial\xi^2} - \frac{1}{4}\,\frac{\partial\Phi_1}{\partial\xi}, \qquad 0<\xi<s(\eta),\\ c_1\,\frac{\partial\theta}{\partial\eta} &= \Lambda_2 N\,\frac{\partial^2\theta}{\partial\xi^2} - \frac{1}{4}\,\frac{\partial\Phi_2}{\partial\xi}, \qquad s(\eta)<\xi<1. \end{aligned} \tag{1}$$

Here $\theta = T/T_r$, $c_i = C_i\rho_i/(C_r\rho_r)$, $\Lambda_i = \lambda_i/\lambda_r$, $\xi = x/L$, $s = S/L$, $\Phi_i = E_i/(\sigma T_r^4)$, $\eta = 4\sigma T_r^3 t/(\rho_r C_r L)$, $N = \lambda_r/(4\sigma T_r^3 L)$ is a radiative-conductive parameter, C, ρ, and λ are the heat capacity, density, and thermal conductivity of the material, respectively, E_i is the resulting radiant flux density, t is time, and σ is the Stefan–Boltzmann constant; the subscript r denotes the determining parameter, and the subscripts $i = 1$ and 2 correspond to the phases on the left and right of the interface $s(\eta)$, respectively.

At the interface, the classical Stefan condition has the form

$$\pm Y\,\frac{ds}{d\eta} = \Lambda_1 N\,\frac{\partial\theta}{\partial\xi}\Big|_{S^-} - \Lambda_2 N\,\frac{\partial\theta}{\partial\xi}\Big|_{S^+} - \frac{1}{4}\Big(\Phi_1\Big|_{S^-} - \Phi_2\Big|_{S^+}\Big), \tag{2}$$

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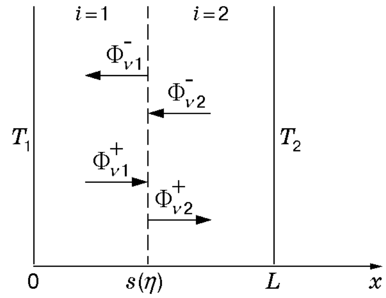

Fig. 1. Two-layer system with a mobile interface.

where $Y = \gamma\rho_{\rm ph}/(\rho_r C_r T_r)$ (γ is the phase-transition heat and $\rho_{\rm ph}$ is the density at the phase-transition temperature).

The temperature at the interface is constant and equal to the phase-transition temperature: $\theta(s(\eta)) = \theta^* = T_{\rm ph}/T_r$.

The initial conditions are written as $\theta(\xi, 0) = f(\xi)$ and $s(0) = 0$.

The solution of the problem consists of determining the temperature $\theta(\xi, \eta)$, the resulting radiant fluxes $\Phi(\xi, \eta)$, and the position of the front $s(\eta)$ in the region $G = \{0 \leqslant \xi \leqslant 1,\ 0 \leqslant \eta \leqslant \eta_1\}$.

The energy equations (1) and (2) are solved by a finite-difference method. The implicit difference scheme is constructed using an integrointerpolation method. The obtained nonlinear system of difference equations is solved by marching and iterative methods. In this case, the radiant fluxes in the energy equation are internal sources and are determined from the solution of the radiative transport equation with known temperature distribution.

In the problem of RCHT in a semitransparent medium with a first-order phase transition, the most difficult problem is to solve the integrodifferential radiative transport equation for a system consisting of two or more layers with a mobile interface. In [4–6], the density resulting radiant flux density was determined from the intensities of forward and backward radiation expressed in terms of formal solutions of radiative transport equations. The integrals included in the formal solution were obtained numerically. This solution is cumbersome because each time when changing the boundary conditions and considering radiation scattering and selectivity, one needs to obtain formal solutions.

There are a great number of approximate methods for solving the radiative transport equation. Modifications of the mean flux method offer ample opportunities for account of scattering anisotropy, boundary reflection of radiation, and radiation selectivity, as applied to radiative and complex heat transfer. Within the framework of this method, the integrodifferential transport equation reduces to a system of two nonlinear differential equations. Among the advantages of this method is the fact that it is easily extended to the case of a selective medium and is applicable to multilayer systems. Rubtsov and Timofeev [7] propose a method for solving the stationary problem of RCHT in a multilayer semitransparent system. Based on the mean flux method, an algorithm for determining radiant fluxes was developed, which yields good accuracy and convergence. In the present paper, the indicated algorithm is used to solve the nonstationary problem of melting and solidification of a semitransparent material to determine radiant fluxes.

The differential analog of the radiative transport equation for hemispherical fluxes in each layer of the system is written as

$$\frac{d}{d\tau_{ji}}(\Phi^+_{ji} - \Phi^-_{ji}) + (1-\omega_{ji})(m^+_{ji}\Phi^+_{ji} - m^-_{ji}\Phi^-_{ji}) = (1-\omega_{ji})n^2_{ji}\Phi_{0ji},$$
$$\frac{d}{d\tau_{ji}}(m^+_{ji}\delta^+_{ji}\Phi^+_{ji} - m^-_{ji}\delta^-_{ji}\Phi^-_{ji}) + (1-\omega_{ji}\bar{\zeta}_{ji})(\Phi^+_{ji} - \Phi^-_{ji}) = 0, \tag{3}$$
$$i = 1, 2, \qquad j = 1, 2, \ldots, J.$$

Here the subscript j denotes the band number in the spectrum with constant values of the optical parameters and the subscript i refers to the relevant layer.

The boundary conditions on the opaque, diffusely radiating, and reflecting surfaces have the form

$$\tau_{j1} = 0: \qquad \Phi^+_{j1} = \varepsilon_{j1}n^2_{j1}\Phi_{0j1}/4 + r_{j1}\Phi^-_{j1},$$
$$\tau_{j2} = \tau_{0j2}: \qquad \Phi^-_{j2} = \varepsilon_{j2}n^2_{j2}\Phi_{0j2}/4 + r_{j2}\Phi^+_{j2}. \tag{4}$$

At the interface, the equation linking the hemispherical radiant fluxes can be obtained from balance relations taking into account boundary reflection and the total internal reflection due to the difference in refractive indices. We set $n_1 = n_2$. Then, on the inner boundary the following condition is satisfied:

$$\tau_{j1} = \tau_{0j1}: \qquad \Phi^{-}_{j1} = \Phi^{-}_{j2}, \quad \Phi^{+}_{j2} = \Phi^{+}_{j1}. \tag{5}$$

In Eqs. (3)–(5), $\Phi^{\pm}_{ji}(\tau_{ji}) = \pm \dfrac{1}{4\sigma T_r^4} \displaystyle\int_{\Delta\nu_j} 2\pi \int_{0(-1)}^{1(0)} I_\nu(\tau_{\nu i}, \mu)\mu \, d\mu \, d\nu$ are the dimensionless densities of the hemispherical fluxes in the band $\Delta\nu_j$, $\Phi_{0ji} = (T_i^4/T_r^4)\varphi_b(\Delta\nu_j)$ is the dimensionless density of the equilibrium radiant flux in the band $\Delta\nu_j$, $\varphi_b(\Delta\nu_j)$ is the Debye function [7], $m^{\pm}_{ji}$ and $\delta^{\pm}_{ji}$ are transport coefficients, which are functionals of the solution and are obtained by iterations, n_{ji} is the spectral refractive index, I_ν is the spectral radiation intensity, μ is the cosine of the angle between the direction of radiation propagation and the x axis, $\tau_{\nu i} = k_{\nu i}x$ is the spectral optical thickness, $k_{\nu i} = \alpha_{\nu i} + \beta_{\nu i}$ is the spectral attenuation factor, $\alpha_{\nu i}$ and $\beta_{\nu i}$ are the spectral absorption and scattering factors, respectively, ε_{ji} and r_{ji} are the emissivity factor and reflectances from opaque surfaces, ω_{ji} is the single scattering albedo, $\bar{\zeta}_{\nu i} = \dfrac{1}{2} \displaystyle\int_{-1}^{1} p_{\nu i}(\mu)\mu \, d\mu$ is the mean spectral cosine of the scattering angle, and $p_{\nu i}$ is the spectral scattering indicatrix, which is the expansion in Legendre polynomials:

$$p(\mu_0) = 1 + \sum_{l=1}^{L} a_l P_l(\mu_0) \tag{6}$$

(μ_0 is the cosine of the angle between the incident and scattered beams).

The mean flux method uses a truncated three-term scattering indicatrix because for $l > 2$, the number of differential equations and unknown functions increase, which complicates the solution of the problem. The proposal to be restricted to three terms in expansion (6) is also justified by the fact that a rather accurate description of the real situation can be achieved in many cases even with a smaller number of expansion terms.

The density of the integral resulting radiation flux is obtained from the relation $\Phi_i = \sum_{j=1}^{J} (\Phi^{+}_{ji} - \Phi^{-}_{ji})$.

Because of the nonlinearity of the radiative problem and the difference in optical thickness between the layers, this problem is solved using the method of iterations. In each layer, the relevant boundary-value problem including system (3) with boundary conditions (4) and (5) is solved. The temperature distribution is considered known. The obtained values of Φ_i are substituted into the energy equations (1) and (2). In the numerical implementation of the algorithm of the mean flux method, the dimensionless coordinates of each layer are varied from 0 to 1. In the solution of the energy equations, the dimensionless thickness of the entire system is also varied from 0 to 1. Thus, the interface $s(\eta)$ is between 0 and 1. The disagreement of the layer thicknesses is resolved as follows. The temperature distribution is first interpolated into the region $[0, 1]$ in each layer, and from known temperatures radiant fluxes are found, which are then interpolated into specified nodes of the calculation grid of the energy equations. The process is then repeated.

Numerical calculations were performed for a hypothetical material, whose properties are close to those of fluorite [$T_{\text{ph}} = 1700$ K, $\lambda_2 = \lambda_r = 9$ W/(m$\cdot$ K), and $L = 0.1$ m]. The dimensionless parameters of the problem for melting are as follows: $c_1 = 0.75$, $c_2 = 1$, $\Lambda_1 = 2$, $\Lambda_2 = 1$, $Y = 0.1$, and $\theta^* = 0.5$. For first-order energy equations, the boundary conditions are $\theta_1 = 0.7$ and $\theta_2 = 0.3$. For solidification, we have $c_1 = c_2 = 1$, $\Lambda_1 = \Lambda_2 = 1$, $Y = 0.1$, and $\theta_1 = 0.3$ ($\partial\theta/\partial\xi = 0$ for $\xi = 1$). For convenience of comparison of the results, the parameters of the problem are same as in [4–6]. The calculations were performed in the gray body approximation with $n_1 = n_2 = 1.5$. The attenuation factors are considered constant over the entire region of frequencies and are determined from the relations $\tau_1 = k_1 L$ and $\tau_2 = k_2 L$.

In [2, 4–6], it is shown that for $N < 0.05$ (when radiation plays a dominant role in heat transfer) there is disturbance of the monotonic nature of temperature distribution ahead of the flat front, which is manifested in overheating of the solid phase during melting or overcooling of the liquid phase during solidification. This causes instability of the phase transition, and in this case, one should use a different model.

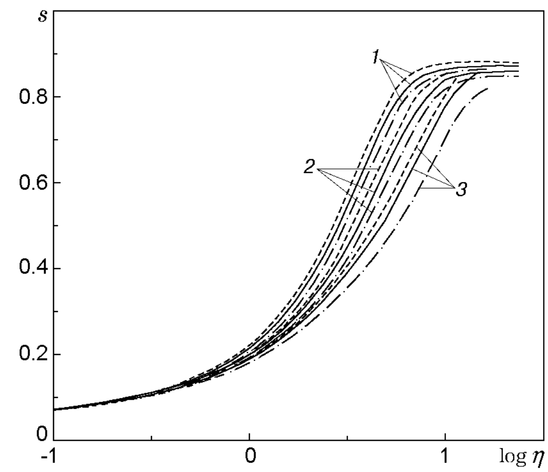

Fig. 2

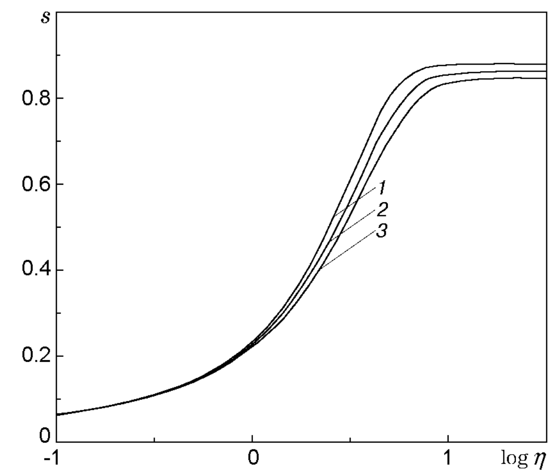

Fig. 3

Fig. 2. Motion of the phase-transition front during melting in a scattering solid phase: solid curves refer to isotropic scattering, dot-and-dashed curves refer to forward scattering ($a_1 = 1.2$ and $a_2 = 0.5$), and dashed curves refer to backward scattering ($a_1 = -1.2$ and $a_2 = 0.5$) for $\omega_1 = 0$ and $\omega_2 = 0.7$ (curves 1), $\omega_1 = 0$ and $\omega_2 = 0.9$ (curves 2), $\omega_1 = 0$ and $\omega_2 = 1.0$ (curves 3).

Fig. 3. Motion of phase-transition front during melting in a scattering liquid phase: curve 1 refers to $\omega_1 = 0.9$, $\omega_2 = 0$, $a_1 = 1.2$, and $a_2 = 0.5$, curves 2 refer to $\omega_1 = 0.9$, and $\omega_2 = 0$, and curves 3 refer to $\omega_1 = 0.9$, $\omega_2 = 0$, $a_1 = -1.2$, and $a_2 = 0.5$.

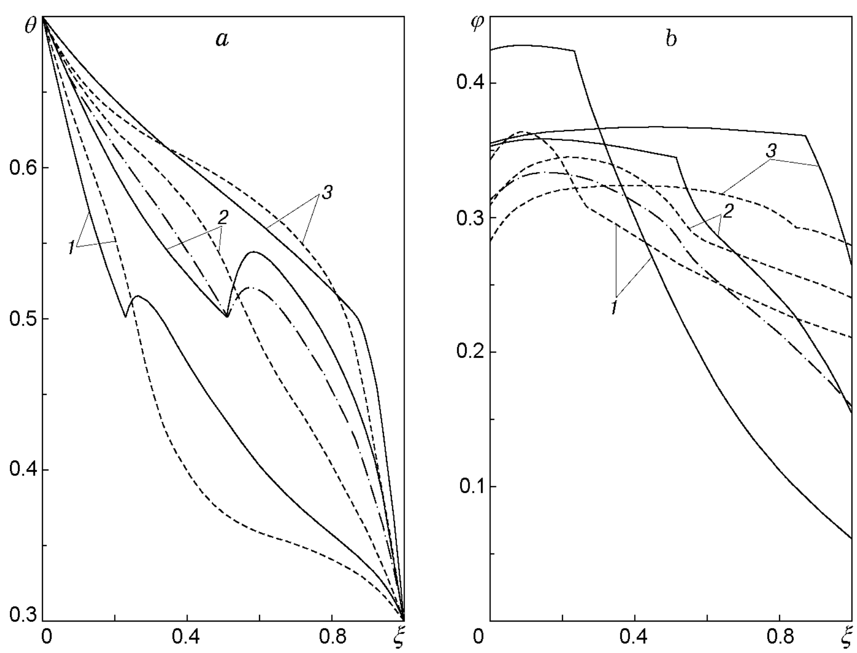

Fig. 4. Temperature distribution (a) and radiant fluxes (b) during melting at various times η: solid curves refer to $\omega_1 = 0.9$, $\omega_2 = 0$, $a_1 = 1.2$, $a_2 = 0.5$, and $\eta = 1$ (1), 2.6 (2), and 10 (3), dashed curves refer to $\omega_1 = 0$, $\omega_2 = 0.9$, $a_1 = 1.2$, $a_2 = 0.5$, and $\eta = 1.63$ (1), 4.76 (2), and 20 (3), and dot-and-dashed curves refer to $\omega_1 = 0.5$, $\omega_2 = 0.5$, and $\eta = 2.95$.

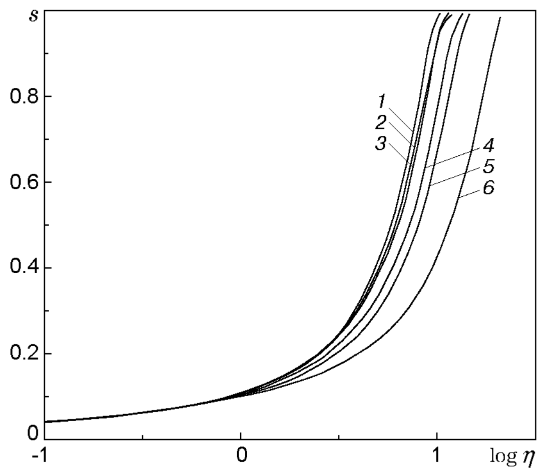

Fig. 5. Effect of isotropic scattering on the rate of solidification: 1) $\omega_1 = 0$ and $\omega_2 = 0$; 2) $\omega_1 = 0.7$ and $\omega_2 = 0$; 3) $\omega_1 = 0.9$ and $\omega_2 = 0$; 4) $\omega_1 = 0.5$ and $\omega_2 = 0.5$; 5) $\omega_1 = 0$ and $\omega_2 = 0.7$; (6) $\omega_1 = 0$ and $\omega_2 = 0.9$.

In the present paper, the classical model is used to study the effect of isotropic and anisotropic scattering of radiation on the rate of motion of the phase-transition front and on the departure of the temperature distribution from a monotonic profile. The absorption factors are $\alpha_1 L = 1$ and $\alpha_2 L = 2$ and the radiative-conductive parameter is $N = 0.01$.

Results of the present calculations of the temperature distribution and radiant fluxes during melting and solidification using the mean flux method are in good agreement with the results of [4–6].

The effect of scattering was analyzed under the assumption of black walls ($r_1 = r_2 = 0$) for various combinations of the single scattering albedo ω_1 and ω_2 corresponding to the different phases. The calculations were performed for the following values of the Legendre polynomial expansion coefficients: $a_1 = 1.2$ and $a_2 = 0.5$ (forward scattering) and $a_1 = -1.2$ and $a_2 = 0.5$ (backward scattering) [8].

The calculation results presented in Fig. 2 show that with increase in the single scattering albedo of the solid phase ω_2 during melting, the velocity of the phase front decreases. Forward scattering in the solid phase decreases and backward scattering increases the rate of melting compared to isotropic radiation. This can be explained by the fact that the energy absorbed by the near-front region decreases due to forward radiation. In contrast, forward scattering in the liquid phase accelerates melting, and backward scattering retards it compared with isotropic radiation (Fig. 3). The radiation scattering in the liquid during melting enhances the temperature "outburst" with increase in ω_1 (Fig. 4), whereas the radiation scattering in the solid phase reduces it. For large values of ω_2 ($\omega_1 = 0$) the temperature distribution becomes monotonic, the radiant flux gradient in the solid phase decreases (Fig. 4), and the rate of melting decreases compared to the rate of melting in the case of a scattering liquid phase (curve 2 in Figs. 2 and 3). With increase in scattering, the radiation absorption by the solid phase in the near-front region decreases, which reduces the temperature "outburst." The results on isotropic radiation agree qualitatively with the results obtained in [2].

Investigation of the role of scattering during solidification showed that the liquid-phase scattering is of significance. With increase in liquid-phase scattering, the rate of solidification decreases considerably compared to the solid-phase scattering (Fig. 5). For $\omega_2 = 1$, the temperature distribution becomes monotonic at any time (dashed curves in Fig. 6a) and the radiant flux gradient in the liquid to the right of the interface decreases to zero (dashed curves in Fig. 6b). For smaller values of ω_2, the nonmonotonic nature of the temperature distribution is preserved but to a variable degree (dot-and-dashed and solid curves in Fig. 6a). For any value of ω_1 (when the solid phase is scattering) the temperature distributions remain nonmonotonic. In this cases, forward scattering accelerates solidification, and backward scattering retards the process compared with isotropic radiation. The effect of anisotropic liquid-phase scattering is insignificant (Fig. 7).

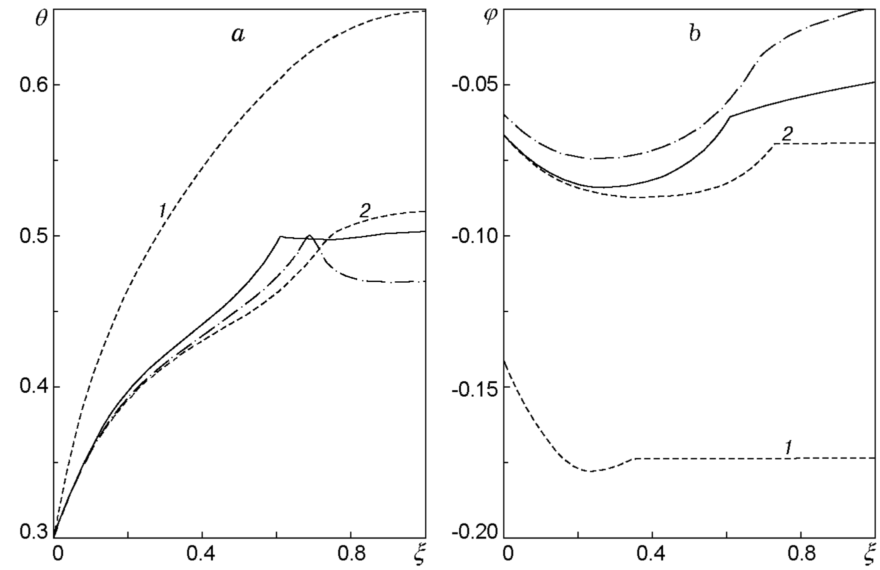

Fig. 6. Effect of scattering on the temperature distribution (a) and radiant fluxes (b) during solidification: dashed curves refer to $\omega_1 = 0$, $\omega_2 = 1$, and $\eta = 15.54$ (1) and 40.63 (2), solid curves refer to $\omega_1 = 0$, $\omega_2 = 0.9$, and $\eta = 14.7$; dot-and-dashed curves $\omega_1 = 0$, $\omega_2 = 0.7$, and $\eta = 10.15$.

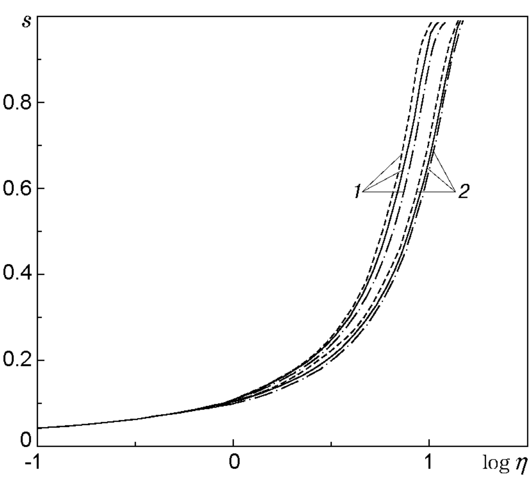

Fig. 7. Effect of anisotropic scattering on the rate of solidification: solid curve refer to isotropic scattering, dashed curves refer to forward scattering ($a_1 = 1.2$ and $a_2 = 0.5$); dot-and-dashed curves refer to backward scattering ($a_1 = -1.2$ and $a_2 = 0.5$) for $\omega_1 = 0.7$ and $\omega_2 = 0$ (curves 1) and $\omega_1 = 0$ and $\omega_2 = 0.7$ (curves 2).

The analysis in [3] of the effect of anisotropic scattering showed that backward scattering retards melting and solidification and forward scattering accelerates these processes. However, in [3], the phase transformation of a semi-infinite semitransparent medium was considered using a generalized model allowing for the formation of a two-phase scattering zone at the phase-transition temperature. Hence, scattering only the second semi-infinite region is scattering.

The results highlight the need for a more correct consideration for the radiation properties related to the optical inhomogeneity on the boundaries and in the volume of systems undergoing phase transitions.

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